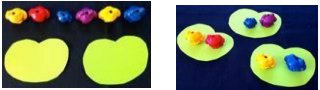
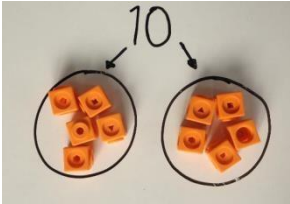
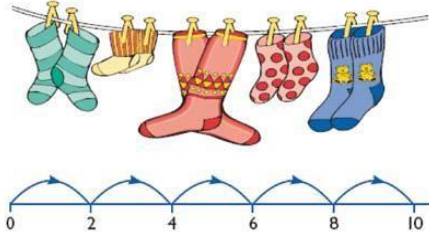
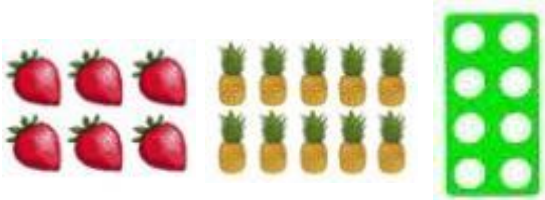

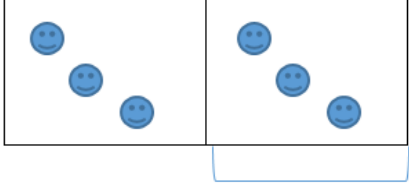




EYFS	
EYFS Early Learning Goals	<p>Number ELG</p> <ul style="list-style-type: none"> • Have a deep understanding of numbers to 10, including the composition of each number. • Recall fluently number bonds up to 5 and some number bonds to 10. • Recognise quantities without counting up to 5. <p>Numerical Patterns ELG Children at the expected level of development will:</p> <ul style="list-style-type: none"> • Count reliably beyond 20, recognising the pattern of the counting system. • Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity. • Explore patterns within numbers to 10, including doubling, halving and sharing.
Year 1	
<p>Basic mathematical vocabulary</p> <p>count in ones, twos... tens... share, groups of, equal groups, odd, even</p> <p>Instructional vocabulary</p> <p>count out, share out, left, left over</p>	
<p>National curriculum link:</p> <p>Solve one – step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.</p>	

Concrete	Pictorial	Abstract
<p>Sharing into equal groups: 6 frogs shared equally between 2 gives 3 frogs on each lily pad or Grouping in equal groups 6 frogs grouped in 2s need 3 lily on</p>  <p>Part Whole Model I have 10 cubes, can you share them equally in 2 groups?</p>  <p>How many 2s</p> 	<p>Arrays:</p> <p>Rectangular arrangements to show equal groups.</p>  <p>Children use pictures or shapes to share quantities</p>  <p>$8 \div 2 = 4$</p> 	<p>Share 9 buns between three people.</p> <p>$9 \div 3 = 3$</p> <p>Decision making (use concrete/pictorial to support the abstract)</p> <p>How many cars can you make if you have 8 wheels?</p>  <p>How many different ways can you arrange 12 buttons in equal groups?</p> 

$$6 \div 2 = 3$$

Year 2

Basic mathematical vocabulary

share, share equally one each, two each, three each... group in pairs, threes... tens equal groups of \div , divide, divided by, divided into left, left over

Instructional vocabulary

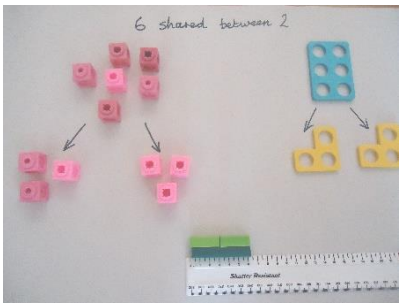
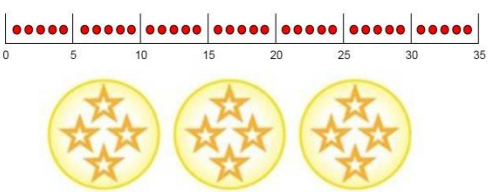
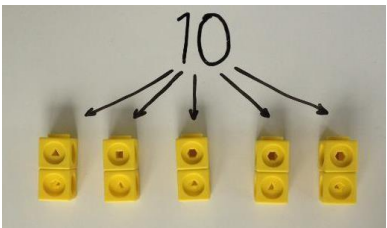



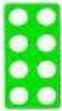
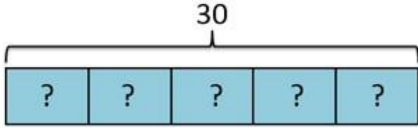
tell me, describe, name, pick out, discuss, talk about, explain, explain your method, explain how you got your answer, give an example of... show how you

National curriculum link:

Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (x), division (\div) and equals (=) signs.

Objectives:

- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
 - Show that the division of one number by another cannot be done in any order (commutative).
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in context.

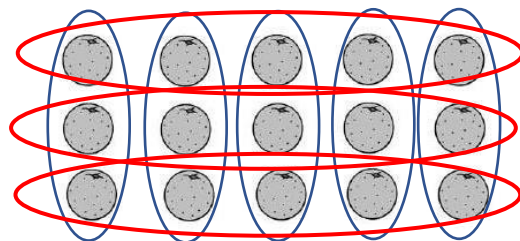
Concrete	Pictorial	Abstract
<p>Divide quantities into equal groups.</p> <p>Use cubes, counters, objects or place value counters to aid understanding.</p>   	<p>Grouping/Sharing models</p> <p>15 frogs shared equally between three lily pads $15 \div 3 = 5$ or 15 frogs grouped in 5s need 3 lily pads to sit on $15 \div 5 = 3$</p>   <p>$15 \div 3 = 5$ (grouping)</p>  <p>$20 \div 10 = 2$</p> <p>Arrays representing the dividend</p> <p>$10 \div 2 = 5$</p> 	<p>$20 \div 5 = 4$</p> <p>Divide 25 into 5 groups. How many are in each group?</p> <p>Find the inverse of multiplication and division sentences by creating four linking number sentences.</p> <p>$7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$</p> <p>Representing problems</p> <p>Jane has 30 cakes. She wants to share them equally between 5 boxes. How many cakes should go in each box?</p>  <p>$30 \div 5 = 6$ Number of cakes in each box = 6</p>

and $10 \div 5 = 2$



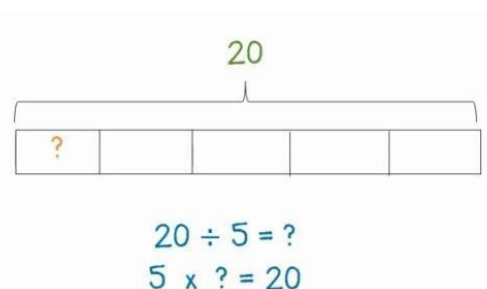
$$15 \div 3 = 5$$

$$15 \div 5 = 3$$



Bar models

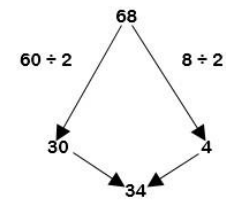
Think of the bar as a whole. Split it into the number you are dividing by and work out how many whole each group



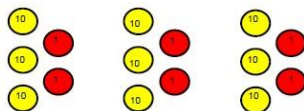
Remainders

The remainder can never be larger than the divisor, eg

$31 \div 5 = 5 \text{ r}6$, as another group of 5 can be made from the r6

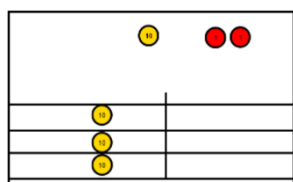
Year 3		
Basic mathematical vocabulary share, share equally one each, two each, three each... group in pairs, threes... tens equal groups of \div , divide, division, divided by, divided into left, left over, remainder, dividend, divisor Instructional vocabulary calculate, work out, solve, investigate, question, answer, check		
National curriculum link: Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods. Objectives: <ul style="list-style-type: none"> Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables. Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects. 		
Concrete	Pictorial	Abstract
Use place value counters to build the dividend (in this example this is 96).	Use pictorial representations of place value counters to build then divide the dividend.	Partitioning strategy to halve Halve 68 Rearranging the dividend to find multiples of the divisor.  $48 \square 3 =$

$$96 \div 3 = 32$$



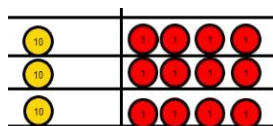
Sharing using place value counters.

$$42 \div 3 = 14$$



1. Make 42. Share the 4 tens between 3. Can we

make an exchange with the extra 10?



Exchange the ten for 10 ones and share out 12 ones



Using known facts

If $3 \times 2 = 6$, then $30 \times 2 = 60$, $60 \div 3 = 20$ and $30 = 60 \div 2$.

Relationships between multiplication, division and fractions

18		
6	6	6

18					
3	3	3	3	3	3

$$6 \times 3 = 18$$

$$18 \div 6 = 3$$

$$\frac{1}{6} \text{ of } 18 = 3$$

$$3 \times 6 = 18$$

$$18 \div 3 = 6$$

$$\frac{1}{3} \text{ of } 18 = 6$$

21						
3	3	3	3	3	3	3

21		
7	7	7

$$3 \times 7 = 21$$

$$21 \div 3 = 7$$

$$\frac{1}{3} \text{ of } 21 = 7$$

$$7 \times 3 = 21$$

$$21 \div 7 = 3$$

$$\frac{1}{7} \text{ of } 21 = 3$$

'What do I know about the 3 x tables?'

"I know $3 \times 10 = 30$ and $3 \times 6 = 18$."

$$\begin{array}{r} 30 \quad 18 \\ \downarrow \quad \downarrow \\ 10 \quad 6 \end{array} \quad 48 \div 3 = 16$$

Short Division

Ensure that this is built on enough of the rearranging strategy above to ensure conceptual understanding

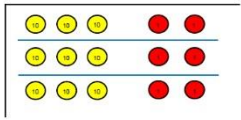


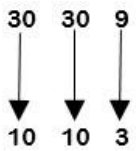
$$\begin{array}{r} 24 \\ 3 \overline{) 72} \end{array}$$


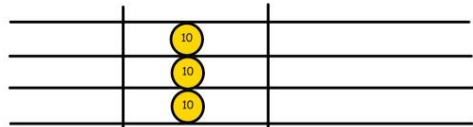
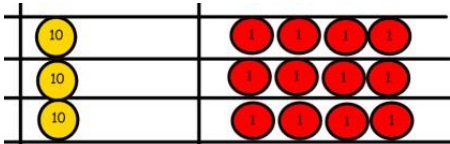
'72 divided by 3. 7 tens shared equally between 3 is 2 with a remainder of 1 ten. Exchange the 1 ten for 10 units. I now have 12 units which shared equally between 3 is 4. The answer is 24.'

Representing problems

Andy says 'I can use my three times table to work out $180 \div 3$ '. Explain what Andy could do to work out this calculation.

		Remainders Complete written divisions and show the remainder using r. $ \begin{array}{ccccccc} 29 & \div & 8 & = & 3 & \text{REMAINDER} & 5 \\ \uparrow & & \uparrow & & \uparrow & & \uparrow \\ \text{dividend} & & \text{divisor} & & \text{quotient} & & \text{remainder} \end{array} $
Year 4		
Basic mathematical vocabulary share, share equally one each, two each, three each... group in pairs, threes... tens equal groups of \div , divide, division, divided by, divided into left, left over, remainder, dividend, divisor		
Instructional vocabulary calculate, work out, solve, investigate question, answer, check		
National curriculum link: To become fluent in the written method of short division.		
Objectives: <ul style="list-style-type: none"> Recall multiplication and division facts for multiplication tables up to 12 x 12 Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers. 		

Concrete	Pictorial	Abstract
<p>Links to tables</p> <p>For example, use language of division linked to tables using counting stick</p> <p>$96 \div 3 = 32$</p> <div> <div>Tens</div> <div>Units</div> <div>3</div> <div>2</div> <div>  </div> </div> <p>Use place value counters to divide using the bus stop method alongside</p> <p>$42 \div 3 =$</p> <div> <div>  </div> <div> <div>Calculations</div> <div>$42 \div 3$</div> <div>  </div> </div> <p>Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.</p> </div>	<p>Using known facts</p> <p>If $2 \times 3 = 6$ then $200 \times 3 = 600$ and $600 \div 3 = 200$</p> <p>Encourage them to move towards counting in multiples to divide more efficiently.</p> <p>Continue to develop rearranging the dividend to find multiples of the divisor.</p> <p>$69 \div 3 =$ 'What do I know about the 3 x tables?' "I know $3 \times 10 = 30$ and $3 \times 3 = 9$."</p> <div> <div>30</div> <div>30</div> <div>9</div> <div>  </div> </div> <p>$69 \div 3 = 23$</p>	<p>Short division</p> <div> <div>2 1 8</div> <div>3</div> <div>4</div> <div>8 7 2</div> </div> <p>Begin with divisions that divide equally with no remainder.</p> <p>$372 \div 6 =$</p> <div> <div>6 2</div> <div>6</div> <div>3 7 2</div> </div> <p>372 divided by 6. 3 hundreds cannot be shared equally between 6, so exchange the hundreds for 30 tens. I now have 37 tens which shared equally between 6 is 6 with a remainder of 1 ten. Exchange the ten for 10 units. I now have 12 units which shared equally between 6 is 2. The answer is 62." Representing problems Alan says that the solution to $186 \div 4$ can be written as '46 remainder 2' or as '46.5'. Do you agree? Explain your answer. Move onto divisions with a remainder</p> <p>Representing problems</p> <p>Alan says that the solution to $186 \div 4$ can be written as '46 remainder 2' or as '46.5'. Do you agree? Explain your answer</p>

<p>We exchange this ten for ten ones and</p>   <p>then share the ones equally among the groups.</p>  <p>We look how much in 1 group so the answer is 14.</p>		<p>Move on to division with remainders</p> $ \begin{array}{r} 24 \text{ r } 1 \\ 3 \overline{) 713} \end{array} $
Year 5		
<p align="center">Basic mathematical vocabulary</p> <p align="center">equal groups of, divide, division, divided by, divided into remainder, factor, quotient, divisible by, inverse</p> <p align="center">Instructional vocabulary</p> <p align="center">calculate, work out, solve, investigate question, answer, check same, different missing number/s number facts, number pairs, number bonds, greatest value, least value</p>		
<p align="center">National curriculum link:</p> <p align="center">Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.</p>		


Objectives:		
<ul style="list-style-type: none"> Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000. Multiply and divide numbers mentally drawing upon known facts. 		
Concrete	Pictorial	Abstract
Refer to Y3/4 concrete materials Go back and use place value counters if children do not understand	Using known facts If $6 \div 2 = 3$ then $6000 \div 2 = 3000$ and $6000 \div 20 = 300$	Short division including interpreting a remainder $484 \div 7 =$ <div style="text-align: right; margin-right: 100px;"> $\begin{array}{r} 69 \text{ r}1 \\ 7 \overline{) 484} \end{array}$ </div> <p>“484 divided by 7. 4 hundreds cannot be shared equally between 7, so exchange the hundreds for 40 tens. I now have 48 tens which shared equally between 7 is 6 with a remainder of 6 tens. Exchange the 6 tens for 60 units, we now have 64 units. 64 shared equally between 7 equals 9 remainder 1. The answer is 69 r1.”</p> <div style="display: flex; justify-content: space-around; align-items: flex-end;"> <div style="text-align: right;"> $\begin{array}{r} 86 \text{ r}2 \\ 5 \overline{) 432} \end{array}$ </div> <div style="text-align: right;"> $\begin{array}{r} 0663 \text{ r}5 \\ 8 \overline{) 5309} \end{array}$ </div> </div> Interpreting remainders $17 \div 5$

"What do I know? 17 is not a multiple of 5."

3 and $\frac{2}{5}$ is equivalent to 3.4 - from knowledge of decimal/fraction equivalents or by converting $\frac{2}{5}$ into $\frac{4}{10}$ which is equivalent to 0.4.

17	$581 \div 7 =$

$581 \div 7$ could be calculated by the formal written method of short division or it could be calculated by rearranging the dividend, using known facts, into 560 and 21.

		Representing problems Correct the errors in the calculation below. Explain the error. $266 \div 5 = 73.1$ 
Year 6		
Basic mathematical vocabulary equal groups of, divide, division, divided by, divided into remainder, factor, quotient, divisible by, inverse, remainders as fractions or decimals Instructional vocabulary calculate, work out, solve, investigate, question, answer, check, same, different, missing number/s, number facts, number pairs, number bonds, greatest value, least value		
National curriculum link: Divide numbers up to 4 digits by a two-digit number using the formal written method of short or long division where appropriate. Objectives: <ul style="list-style-type: none"> • Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. • Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context. <ul style="list-style-type: none"> • Perform mental calculations, including with mixed operations and large numbers. • Use their knowledge of the order of operations (BODMAS) to carry out calculations involving the four operations. 		

Abstract

Using known facts

If $6 \div 2 = 3$ then $6 \div 0.2 = 30$ and $6 \div 0.02 = 300$

Rearranging the dividend to find multiples of the divisor.

$$581 \div 7 =$$

$$560 + 21 =$$

$$58180 + 3 = 83$$

Encourage them to move towards counting in multiples to divide more efficiently.

Short division

$$97.6 \div 5 =$$

$$\begin{array}{r} 19.52 \\ 5 \overline{) 947.2610} \end{array}$$

"97.6 divided by 5. 9 tens shared equally between 5 is 1 with a remainder of 4 tens. Exchange the ten for 10 units. I now have 47 units which shared equally between 5 is 9 with a remainder of 2 units. Exchange the 2 units for 20 tenths, we now have 26 tenths. 26 shared equally between 5 equals 5 with a remainder of 1 tenth. Extend the dividend with a 0 in the hundredths column. Exchange the tenth for 10 hundredths. 10 shared equally between 5 equals 2. The answer is 19.52."

Children are encouraged to create a 'WIK' for long division problems to support them. Children use their knowledge of repeated addition to create a WIK (What I know) for example:

$$\begin{array}{r}
 291 \\
 45 \overline{)13095} \\
 \underline{90} \\
 409 \\
 \underline{405} \\
 45 \\
 \underline{45} \\
 0
 \end{array}$$

WIK

45

90

135

180

225

270

315

360

405

450

With questions of this type where the divisor is close to a number linked to the times tables, encourage the children to use known facts.

Representing problems

Megan divides 500 by 8 and gets the answer $62r4$. She re writes it as $62 \text{ r } 1/2$. Is she right? Explain your answer.

Simplify the fractions for remainders

long division

$$560 \div 24$$

$$\begin{array}{r} 23 \text{ r } 8 \\ 24 \overline{) 560} \\ \underline{-48} \\ 80 \\ \underline{-72} \\ 8 \end{array}$$

remainder as a
whole number

$$\begin{array}{r} 23 \text{ } 8/24 (1/3) \\ 24 \overline{) 560} \\ \underline{-48} \\ 80 \\ \underline{-72} \\ 8 \end{array}$$

remainder as a
fraction in its
lowest form

$$\begin{array}{r} 23.3 \\ 24 \overline{) 560.0} \\ \underline{-48} \\ 80 \\ \underline{-72} \\ 80 \\ \underline{-72} \\ 8 \end{array}$$

remainder
as a decimal